Markov

$X \geq 0 \Rightarrow P(X \geq \alpha) \leq \frac{\mathbb{E}X}{\alpha}$

Cebicev

$P(|X - \mathbb{E}X| > k\sigma) \leq \frac{1}{k^2} \quad k \geq 1 \; ; \quad P(|X| > c) \leq \frac{\mathbb{E}X^2}{c^2} \quad c > 0$

$P(X = x) = \sum_{y \in Y(\Omega)} P(X = x | Y = y) P(Y = y)$

$P(X+Y = z) = \sum_{y \in Y(\Omega)} P(X = z - y) P(Y = y)$

Convoluzione

$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y) \, dy$

Funzione Rischio

$X > 0 \quad h(x) = \frac{f(x)}{S(x)} \quad S(x) = e^{-\int_0^x h(t)dt}$

$P(X \leq x + \Delta x \,|\, X > x) \simeq \Delta x \, h(x)$

h crescente $\Rightarrow$ invecchiamento

1) $h(x) \geq 0$
2) $\int_0^{\infty} h(x)dx = +\infty$

Bayes

$P(E|H_r)$ verosim.

$\frac{P(H_r|E)}{P(H_r)} = \max_r \frac{P(H_r|E)}{P(H_r)} \Leftrightarrow H_r$ è la più verosim.

$P(H_r|E) = \max_r P(H_r|E)$ Hp che diventa più probab.

$\sum_{i=1}^{n} x^{i-1} = \frac{1-x^n}{1-x}$

Binomiale

$p_k = \binom{n}{k} p^k (1-p)^{n-k}$ $k = 0, 1, \dots, n$ $\mathbb{E}X = np$, $\mathrm{Var}X = npq$

k teste in n lanci $X = \#$ teste

Uniforme

$U[a,b] \Rightarrow \mathbb{E}X = \frac{a+b}{2}$ $\mathrm{Var}X = \frac{(b-a)^2}{12}$

Poisson

$p_k = e^{-\lambda}\frac{\lambda^k}{k!}$ $\lambda > 0$ $\mathbb{E}X = \lambda$; $\mathrm{Var}X = \lambda$

k teste in $n = \infty$ lanci $X + Y \sim \mathrm{Pois}(\lambda_1 + \lambda_2)$; $P(X = x \mid X + Y = n) \sim \mathrm{Binom}\left(n, \frac{\lambda_1}{\lambda_1 + \lambda_2}\right)$ X, Y indip.

Geometrica

$p_n = p(1-p)^{n-1}$ $\mathbb{E}X = \frac{1}{p}$ $\mathrm{Var} = \frac{q}{p^2}$; $X = \#$ lanci; $P(X \geq n) = q^{n-1}$

testa dopo n lanci; assenza memoria $P(X > x_0 + x \mid X > x_0) = P(X > x_0)$

Pascal

$P(X = n) = \binom{n-1}{k-1} p^k (1-p)^{n-k}$ $\mathbb{E}X = \frac{k}{p}$ $\mathrm{Var}X = \frac{kq}{p^2}$

k teste dopo n lanci

$X_i = \#$ lanci per ottenere l'i-esima testa; $X = \sum_{i=1}^{k} X_i$

Esponenziale

$f(x) = \lambda e^{-\lambda x}$ $x \geq 0$ $\lambda > 0$ $F(x) = 1 - e^{-\lambda x}$ $\mathbb{E}X = \frac{1}{\lambda}$; $\mathrm{Var} = \frac{1}{\lambda^2}$;

$f(x) = 0$ $x < 0$

assenza di memoria

$\mathbb{E}X^2 = \frac{2}{\lambda^2}$

$\min(X, Y) \sim \mathrm{Exp}(\lambda_1 + \lambda_2)$ $[X, Y$ indip$]$

$Z = \max(X, Y) \Rightarrow P(Z \leq z) = P(X \leq z)P(Y \leq z)$

Normale

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$ $\mathbb{E}X = m$ $\mathrm{Var}X = \sigma^2$

$\Phi(x) = 1 - \Phi(-x)$

$\Phi(x) - \Phi(-x) = 2\Phi(x) - 1$

$\Phi(0) = \frac{1}{2}$

$Y = aX + b \sim \mathrm{Norm}(am + b, |a|\sigma)$

$P(|X - m| \leq k\sigma) = 2\Phi(k) - 1$

Hypergeom

$p_k = \frac{\binom{m_1}{k}\binom{m_2}{n-k}}{\binom{m_1+m_2}{n}}$ $\mathbb{E}X = np$, con $p = \frac{m_1}{m_1+m_2} = P(E_i)$

$\mathrm{Var}X = np(1-p)\left(\frac{N-n}{N-1}\right)$ con $N = m_1 + m_2$

senza restituzione

$X = |E_1| + \dots + |E_n|$; E_i sono scambiabili

Multinomiale

$\{E_1^{(i)}, \dots, E_m^{(i)}\}$ $P(X_1 = x_1, \dots, X_m = x_m) = \binom{n}{x_1, \dots, x_m} p_1^{x_1} \cdots p_m^{x_m}$

$x_1 + \dots + x_m = n$

$X_i = x_i$ $P(X = x_i) = \binom{n}{x_i} p_i^{x_i} (1-p_i)^{n-x_i}$

$X_k = \sum_{i=1}^{n} |E_k^{(i)}|$