

Prop 5.3.5

Sia $F \in P^n$ e sia A_i l' i -esimo punto fondamentale, ovvero $A_{ij} = \delta_{ij}$, allora

$m_{A_i} F = s \iff$ la massima potenza di x_i che compare in F e' $m-s$ con $m = \deg F$

pf $\Rightarrow$

$$F \text{ omogenea} \Rightarrow F = \sum_{j=0}^m x_i^{m-j} F_j \quad F_j \in K[x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{n+1}]$$

(1)

$\deg F_j = j, F_j \text{ omogenea oppure } F_j \text{ nullo}$

2 retta: $A_i \in r$

$A_{ii} = 1 \Rightarrow r \notin \{x_i = 0\} \Rightarrow \exists! B \in r \cap \{x_i = 0\}, B = [b_1, \dots, b_{i-1}, 0, b_{i+1}, \dots, b_{n+1}]$

$G \in H$

La retta r ha equazioni $X = \lambda A_i$ + ~~hp~~ $\Rightarrow$

Sia $B' = (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_{n+1})$

$F_j \in K[x_1, \dots, x_i, \dots, x_{n+1}] \Rightarrow F_j(A_i + \mu B) = F_j(\mu B')$

$$\begin{cases} x_i = \mu b_i \\ \vdots \\ x_{i-1} = \mu b_{i-1} \\ x_i = \lambda \\ x_{i+1} = \mu b_{i+1} \\ \vdots \\ x_{n+1} = \mu b_{n+1} \end{cases}$$

F_j omog.

$$F(A_i + \mu B) = \sum_{j=0}^m \lambda^{m-j} F_j(A_i + \mu B) = \sum_{j=0}^m \lambda^{m-j} F_j(\mu B') = \sum_{j=0}^m \lambda^{m-j} \mu^j F_j(B') = \sum_{j=0}^m \lambda^{m-j} \mu^j F_j(B') \quad (2)$$

$F(A + \mu B) = \sum \mu^j F_j(B') \quad (2')$

$m_{A_i} F = s \iff m_{A_i} F(A_i + \mu B) = s \Rightarrow \mu^s / F(A_i + \mu B) \Rightarrow \left. \begin{matrix} \deg F_j = j \\ (2)' \end{matrix} \right\} \Rightarrow F_j(B') = 0 \quad \forall j < s$

$F_j(B') = 0 \Rightarrow$ questo vale $\forall$ retta t.c. $A_i \in r$ e quindi $\forall B'$

$\Rightarrow F_j \equiv 0 \Rightarrow x_i^{m-s}$ e' la piu' grande potenza di x_i che compare in F

(1)

pf $\Leftarrow$

Per hp $F_0 = F_1 = \dots = F_{s-1} \equiv 0, F_s \neq 0$

$\exists B': F_s(B') \neq 0 \Rightarrow \mu^s / F \Rightarrow m_{A_i} F(A_i + \mu B) = s$

$m_{A_i} F = s$

oss 5.3.6

Sia $F \in P^n$ oppure $F \in A^n$, $\deg F = m$

F e' un cono di vertice $A \iff m_A F = \deg F$

pf

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